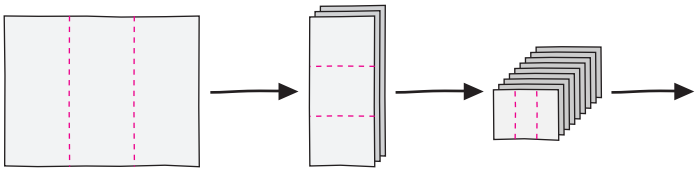
Applications Connections Extensions

ACE

Applications

1. Cut a sheet of paper into thirds. Stack the three pieces and cut the stack into thirds. Stack all the pieces and cut the stack into thirds again.



a. Copy and complete this table to show the number of ballots after each of the first five cuts.

Number of Cuts	Number of Ballots
1	3
2	■
3	■
4	■
5	■

b. Suppose you continued this process. How many ballots would you have after 10 cuts? How many would you have after n cuts?

c. How many cuts would it take to make at least one million ballots?

Write each expression in exponential form.

2. $2 \times 2 \times 2 \times 2$

3. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

4. $2.5 \times 2.5 \times 2.5 \times 2.5 \times 2.5$

Write each expression in standard form.

5. 2^{10} **6.** 10^2 **7.** 3^9

8. You know that $5^2 = 25$. Use this fact to evaluate 5^4.

9. The standard form for 5^{14} is 6,103,515,625. Use this fact to evaluate 5^{15}.

10. Multiple Choice Which expression is equal to one million?

A. 10^6 **B.** 6^{10} **C.** 100^2 **D.** 2^{100}

11. Use exponents to write an expression for one billion (1,000,000,000).

Decide whether each number is greater or less than one million *without using a calculator.* Try to decide without actually multiplying. Explain how you found your answer. Use a calculator to check whether you are right.

12. 9^6 **13.** 3^{10} **14.** 11^6

For Exercises 15–20, write the number in exponential form using 2, 3, 4, or 5 as the base.

15. 125 **16.** 64 **17.** 81

18. 3,125 **19.** 1,024 **20.** 4,096

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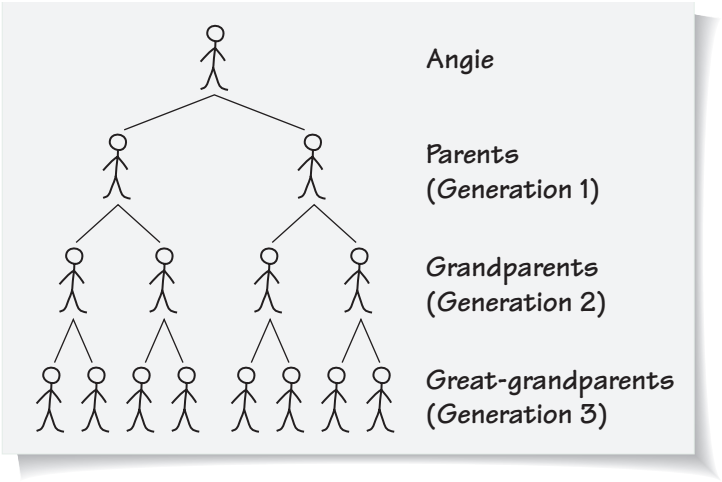
21. While studying her family's history, Angie discovers records of ancestors 12 generations back. She wonders how many ancestors she has had in the past 12 generations. She starts to make a diagram to help her figure this out. The diagram soon becomes very complex.



a. Make a table and a graph showing the number of ancestors in each of the 12 generations.

b. Write an equation for the number of ancestors a in a given generation n.

c. What is the total number of ancestors in all 12 generations?

22. Many single-celled organisms reproduce by dividing into two identical cells. Suppose an amoeba (uh MEE buh) splits into two amoebas every half hour.

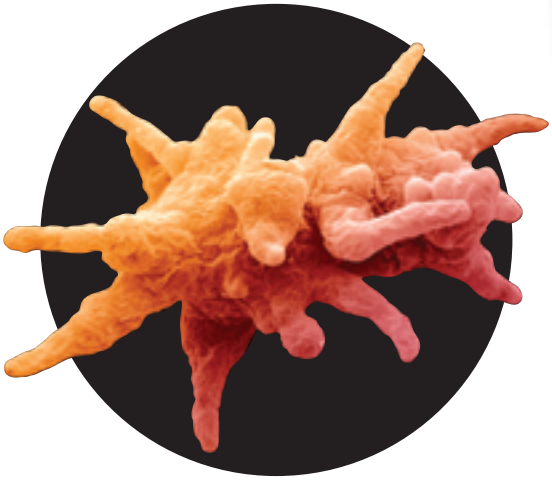
a. An experiment starts with one amoeba. Make a table showing the number of amoebas at the end of each hour over an 8-hour period.

b. Write an equation for the number of amoebas a after t hours.

c. After how many hours will the number of amoebas reach one million?

d. Make a graph of the (*time, amoebas*) data from part (a).

e. What similarities do you notice in the pattern of change for the number of amoebas and the patterns of change for other problems in this investigation? What differences do you notice?



23. Zak's wealthy uncle wants to donate money to Zak's school for new computers. He suggests three possible plans for his donations.

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Plan 1: He will continue the pattern in this table until day 12.

Day	1	2	3	4
Donation	$1	$2	$4	$8

Plan 2: He will continue the pattern in this table until day 10.

Day	1	2	3	4
Donation	$1	$3	$9	$27

Plan 3: He will continue the pattern in this table until day 7.

Day	1	2	3	4
Donation	$1	$4	$16	$64

a. Copy and extend each table to show how much money the school would receive each day.

b. For each plan, write an equation for the relationship between the day number n and the number of dollars donated d.

c. Which plan would give the school the greatest total amount of money?

d. Zak says there is more than one equation for the relationship in Plan 1. He says that $d = 2^{n-1}$ and $d = \frac{1}{2}(2^n)$ both work. Is he correct? Are there two equations for each of the other plans?

24. Jenna is planning to swim in a charity swim-a-thon. Several relatives said they would sponsor her. Each of their donations is explained.

Grandmother: I will give you $1 if you swim 1 lap, $3 if you swim 2 laps, $5 if you swim 3 laps, $7 if you swim 4 laps, and so on.

Mother: I will give you $1 if you swim 1 lap, $3 if you swim 2 laps, $9 if you swim 3 laps, $27 if you swim 4 laps, and so on.

Aunt Lori: I will give you $2 if you swim 1 lap, $3.50 if you swim 2 laps, $5 if you swim 3 laps, $6.50 for 4 laps, and so on.

Uncle Jack: I will give you $1 if you swim 1 lap, $2 if you swim 2 laps, $4 if you swim 3 laps, $8 if you swim 4 laps, and so on.

a. Decide whether each donation pattern is *exponential*, *linear*, or *neither*.

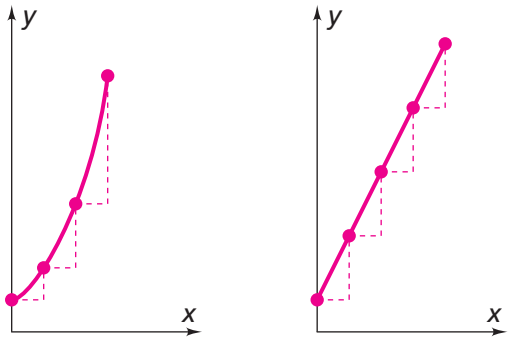
b. For each relative, write an equation for the total donation d if Jenna swims n laps.

c. For each plan, tell how much money Jenna will raise if she swims 20 laps.

25. The graphs below represent $y = 2^x$ and $y = 2x + 1$.

a. Tell which equation each graph represents. Explain your reasoning.

b. The dashed segments show the vertical and horizontal change between points at equal x intervals. For each graph, compare the vertical and horizontal changes between pairs of points. What do you notice?



Graph 1 Graph 2

Study the pattern in each table.

a. Tell whether the relationship between x and y is *linear, exponential,* or *neither*. Explain your reasoning.

b. If the relationship is linear or exponential, give its equation.

26.

x	0	1	2	3	4	5
y	10	12.5	15	17.5	20	22.5

27.

x	0	1	2	3	4
y	1	6	36	216	1,296

28.

x	0	1	2	3	4	5	6	7	8
y	1	5	3	7	5	8	6	10	8

29.

x	0	1	2	3	4	5	6	7	8
y	2	4	8	16	32	64	128	256	512

30.

x	0	1	2	3	4	5
y	0	1	4	9	16	25

Connections

31. Refer to Problem 1.1. Suppose a stack of 250 sheets of paper is 1 inch high.

a. How high would the stack of ballots be after 20 cuts? How high would it be after 30 cuts?

b. How many cuts would it take to make a stack 1 foot high?

32. In Problem 1.2, suppose a Montarek ruba had the value of a modern U.S. penny. What would be the dollar values of the rubas on squares 10, 20, 30, 40, 50, and 60?

33. A ruba had the same thickness as a modern U.S. penny (about 0.06 inch). Suppose the king had been able to reward the peasant by using Plan 1 (doubling the number of rubas in each square).

a. What would have been the height of the stack of rubas on square 64?

b. The average distance from Earth to the moon is about 240,000 miles. Which (if any) of the stacks would have reached the moon?

34. One of the king's advisors suggested this plan: Put 100 rubas on the first square of a chessboard, 125 on the second square, 150 on the third square, and so on, increasing the number of rubas by 25 for each square.

a. Write an equation for the numbers of rubas r on square n for this plan. Explain the meanings of the numbers and variables in your equation.

b. Describe the graph of this plan.

c. What is the total number of rubas on the first 10 squares? What is the total number on the first 20 squares?

For Exercises 35–37, find the slope and *y*-intercept of the graph of each equation.

35. $y = 3x - 10$ **36.** $y = 1.5 - 5.6x$ **37.** $y = 15 + \frac{2}{5}x$

38. Write an equation whose line is less steep than the line represented by $y = 15 + \frac{2}{5}x$.

39. Sarah used her calculator to keep track of the number of rubas in Problem 1.2. She found that there will be 2,147,483,648 rubas on square 32.

a. How many rubas will be on square 33? How many will be on square 34? How many will be on square 35?

b. Which square would have the number of rubas shown here?

2,147,483,648 · 2 · 2 · 2 · 2 · 2 · 2 · 2 · 2 · 2

c. Use your calculator to do the multiplication in part (b). Do you notice anything strange about the answer your calculator gives? Explain.

d. Calculators use shorthand notation for showing very large numbers. For example, if you enter 10^12 on your calculator, you may get the result 1E12. This is shorthand for the number 1.0×10^{12}. The number 1.0×10^{12} is written in **scientific notation.** For a number to be in scientific notation, it must be in the form:

(*a number greater than or equal to 1 but less than 10*) × (*a power of 10*)

Write 2,147,483,648 · 2 · 2 · 2 · 2 · 2 · 2 · 2 · 2 · 2 in scientific notation.

e. Write the numbers 2^{10}, 2^{20}, 2^{30}, and 2^{35} in both standard and scientific notation.

f. Explain how to write a large number in scientific notation.

Write each number in scientific notation.

40. 100,000,000 **41.** 29,678,900,500 **42.** 11,950,500,000,000

Find the largest whole-number value of *n* for which your calculator will display the result in standard notation.

43. 3^n **44.** π^n **45.** 12^n **46.** 237^n

Extensions

47. Consider these two equations:

Equation 1: $r = 3^n - 1$ **Equation 2:** $r = 3^{n-1}$

a. For each equation, find r when n is 2.

b. For each equation, find r when n is 10.

c. Explain why the equations give different values of r for the same value of n.

48. This table represents the number of ballots made by repeatedly cutting a sheet of paper in half four times. Assume the pattern continues.

Number of Cuts	Number of Ballots
1	2
2	4
3	8
4	16

a. Write an equation for the pattern in the table.

b. Use your equation and the table to determine the value of 2^0.

c. What do you think b^0 should equal for any number b? For example, what do you think 6^0 and 23^0 should equal? Explain.

49. When the king of Montarek tried to figure out the total number of rubas the peasant would receive under Plan 1, he noticed an interesting pattern.

a. Extend and complete this table for the first 10 squares.

Reward Plan 1

Square	Number of Rubas on Square	Total Number of Rubas
1	1	1
2	2	3
3	4	7
4	■	■

b. Describe the pattern of growth in the total number of rubas as the number of the square increases.

c. Write an equation for the relationship between the number of the square n and the total number of rubas t on the board.

d. When the total number of rubas reaches 1,000,000, how many squares will have been covered?

e. Suppose the king had been able to give the peasant the reward she requested. How many rubas would she have received?

50. Refer to Plans 1–4 in Problems 1.2 through 1.4.

a. Which plan should the king choose? Explain.

b. Which plan should the peasant choose? Explain.

c. Write an ending to the story of the king and the peasant.